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Constant Altitude–Constant Mach Number Cruise Range of Transport Aircraft with Compressibility Effects

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An approximate solution of the constant altitude–constant Mach number cruise range for high subsonic speed flight of the turbojet/fan aircraft is proposed. The solution considers cambered wing drag polar of modern transport aircraft, dependence of the specific fuel consumption on Mach number, and compressibility effects on aerodynamic characteristics of the aircraft. The method aims for a quick assessment of the cruise range during conceptual or preliminary design phase. An application of the method to a known type of aircraft is also presented.

Nomenclature

AR	=	wing aspect ratio
a	=	speed of sound
a_t	=	speed of sound at the tropopause
C_D	=	drag coefficient
C_{D_c}	=	compressible drag coefficient
$C_{D_{min}}$	=	minimum parasite drag coefficient
C_{D_0}, C'_{D_0}	=	parasite drag coefficient
C_L	=	lift coefficient
C_{L_D}	=	design lift coefficient
$C_{L_{min}}$	=	lift coefficient at zero angle of attack
c_T	=	thrust specific fuel consumption
c_{T_0}	=	thrust specific fuel consumption constant at a given altitude
c_T^*	=	engine manufacturer's reference cruise thrust specific fuel consumption
c_0, c_1	=	thrust specific fuel consumption coefficients
D	=	drag
e	=	Oswald efficiency factor
K, K_1, K_2, K_3, K_4	=	induced drag coefficients
K'_1, K'_2	=	induced drag coefficients
L	=	lift
M	=	Mach number
M_{crit}	=	critical Mach number
M_{DD}	=	drag divergence Mach number
M_{MD}	=	minimum drag Mach number
M^*	=	engine manufacturer's reference cruise Mach number
p	=	ambient air pressure
R	=	range
R_B	=	Bréguet range
R'	=	dimensionless range
S	=	wing area
t/c	=	thickness/chord ratio
V_{MD}	=	minimum drag speed
W	=	aircraft weight
β	=	thrust specific fuel consumption Mach number exponent
Γ	=	compressibility parameter
γ	=	specific heat ratio of air, 1.4
ζ	=	fuel weight fraction
θ	=	relative temperature
θ_t	=	relative temperature at tropopause
θ^*	=	relative temperature at the engine manufacturer's reference cruise altitude
κ_a	=	airfoil technology factor
Λ	=	wing sweep angle at quarter chord

Subscripts

cr	=	cruise
c	=	compressible
f	=	fuel
0	=	initial condition

Introduction

RANGE is a basic operational requirement and design criterion especially for transport category airplanes. Because of this, range performance has been studied by several researchers. In the classical analytical approach, the optimum cruise of a jet aircraft is usually considered with some simplifications such as no compressibility effects, a constant thrust specific fuel consumption,[1] the constant speed-constant lift coefficient cruise (Breguet range), and a simple parabolic drag polar. Recently, Cavcar and Cavcar[2] proposed approximate constant altitude–constant Mach number cruise range solutions considering a parabolic drag polar for cambered wing and variation of the thrust specific fuel consumption with the Mach number. However, compressibility effects were neglected in Ref. 2. Cavcar and Cavcar[3] attempted to present another approximate solution to include compressibility effects into solution of the problem by assuming that the cruise lift coefficient varies around $C_L = 0.5$. In this paper, a more generalized approximate solution is presented considering 1) variation of the thrust specific fuel consumption with the altitude and Mach number,[1] 2) modern transport aircraft cambered wing with compressibility effects,[3] and 3) constant altitude–constant Mach number cruise implied by air traffic control.[2]

Therefore, all major conventional assumptions are removed, and an analytical solution is attempted. To demonstrate the accuracy of the solution, an application of the method to Boeing 747 aircraft is also presented.

Problem Formulation

Thrust specific fuel consumption

Martinez-Val and Perez[1] and Torenbeek[4] state that the thrust specific fuel consumption varies linearly with the Mach number, that is,

$$c_T = c_0 + c_1 M \tag{1}$$

However, Martinez-Val and Perez proved that the thrust specific fuel consumption assumption

$$c_T = c_T^* (M/M^*)^\beta \sqrt{\theta/\theta^*} \tag{2}$$

provides a highly accurate, but also a more simple solution, for cruise range calculations. Later Ananthasayanam[5] gave the same relation

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for the thrust specific fuel consumption. In Eq. (2) terms with a superscript asterisk represent the cruise specific fuel consumption, the Mach number, and relative temperature of the cruising altitude cited by the engine manufacturer. Here β is the exponent in the dependence of thrust specific fuel consumption with M, and the value of β is on the order of 0.2–0.4 for low bypass turbofan engines and 0.4–0.7 for high bypass turbofan engines. Shevell[6] defined the turbofan engines with bypass ratios of 5–6 as high-bypass-ratio turbofan engines. According to Torenbeek,[7] engines with bypass ratios less than 2 are low-bypass-ratio engines, and engines with bypass ratios over 2 are high-bypass-ratio turbofan engines.

Validity of the value of β as given by Martinez-Val and Perez[1] and Ananthasayanam[5] is checked by its application to two different types of turbofan engines, Pratt and Whitney JT9D-7A (Ref. 8), with a bypass ratio of 5.1, and Pratt and Whitney Canada JT15D-4C (Ref. 8), with a bypass ratio of 2.7. For the JT9D-7A engine, the performance data presented by McCormick[8] are used. The JT15D-4C engine performance data were obtained from the Pratt and Whitney Canada engine brochure.[9] As shown in Fig. 1, the value of β is approximately 0.44 at pressure altitudes of 35,000 and 40,000 ft for the JT9D-7A engine. The higher value of 0.47 is obtained at the lower 30,000 ft pressure altitude. The value of β is found to be approximately 0.29 for the JT15D-4C engine as shown in Fig. 2. These two applications indicate suitability of the β values proposed by Martinez-Val and Perez[1] and Ananthasayanam.[5]

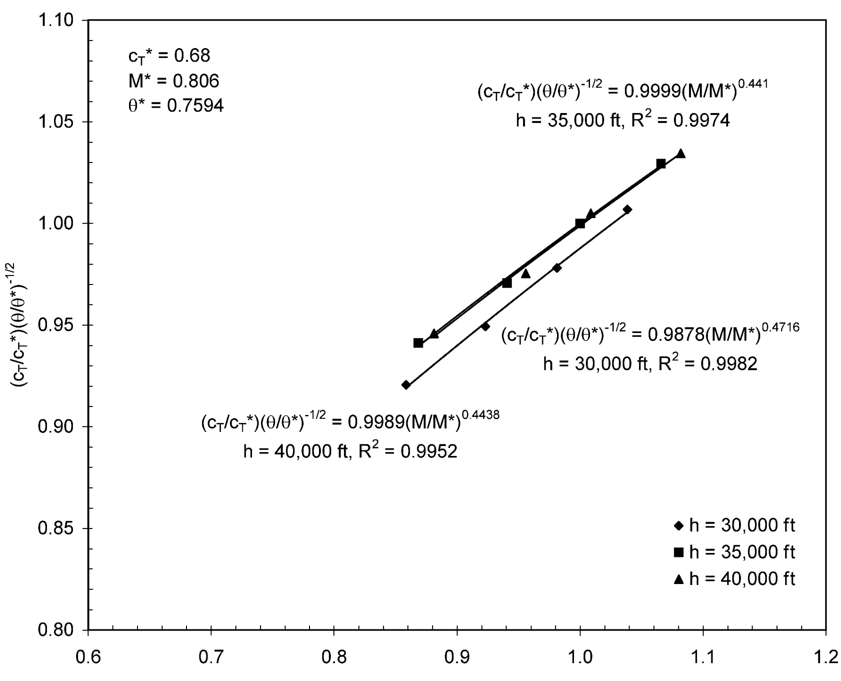

Fig. 1 Cruise thrust specific fuel consumption of JT-9D-7A turbofan engine (data source, McCormick[8]).

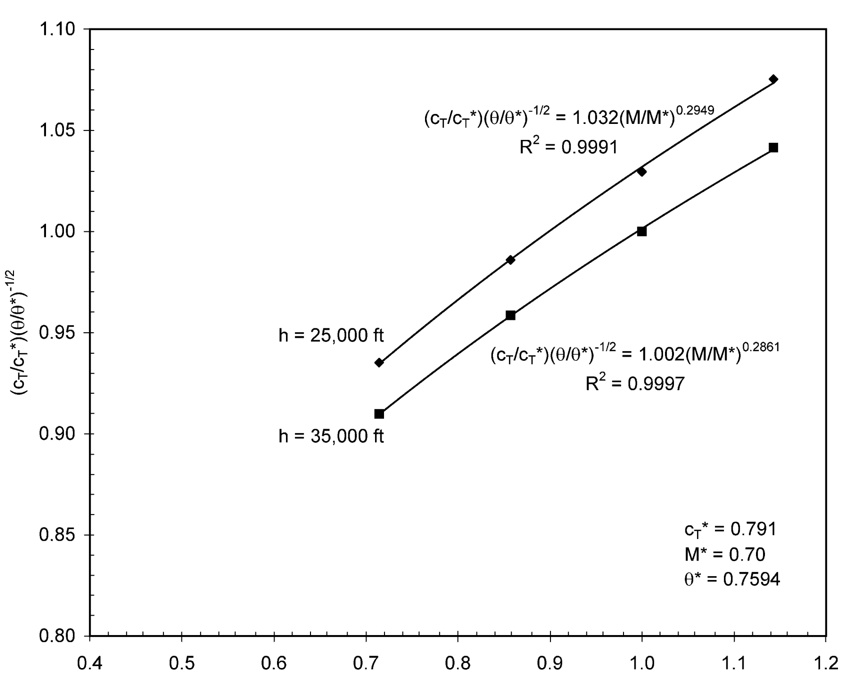

Fig. 2 Cruise thrust specific fuel consumption of JT-15D-4C turbofan engine (data soruce, Pratt and Whitney Canada[9]).

For constant altitude cruise, the thrust specific fuel consumption given by Eq. (2) can be written

$$c_T = c_{T_0} M^{\beta} \sqrt{\theta} \tag{3}$$

where

$$c_{T_0} = c_T^* \Big/ (M^*)^{\beta} \sqrt{\theta^*} \tag{4}$$

Aerodynamic Model

Modern transport aircraft cruise in transonic flow conditions, usually around the drag divergence Mach number. Thus, the drag polar must be

$$C_D = C_{D_{\min}} + K\left(C_L - C_{L_{\min}}\right)^2 + \Delta C_{D_C} \tag{5}$$

where induced-drag coefficient K and compressible drag rise ΔC_{D_c} are functions of the Mach number. For subcritical Mach numbers,

$$K = 1 \Big/ \pi A\!\!R e \sqrt{1 - M^2} \tag{6}$$

and for $M > M_{\text{crit}}$,

$$K = 1 \Big/ \pi A\!\!R e \sqrt{1 - M_{\text{crit}}^2} \tag{7}$$

For subcritical Mach numbers, $\Delta C_{D_C} = 0$. For $M > M_{\text{crit}}$, Grasmeyer,[10] citing Hilton,[11] gave the following empirically derived drag rise function proposed by Lock:

$$\Delta C_{D_C} = 20(M - M_{\text{crit}})^4 \tag{8}$$

For the assumption that $\partial C_D / \partial M = 0.1$ at the drag divergence Mach number from Eq. (8), the critical Mach number becomes

$$M_{\text{crit}} = M_{\text{DD}} - \sqrt[3]{0.1/80} = M_{\text{DD}} - 0.1077 \tag{9}$$

or for the assumption that the drag divergence occurs at $\Delta C_{D_c} = 0.002$,

$$M_{\text{crit}} = M_{\text{DD}} - 0.1 \tag{10}$$

Both assumptions for the drag divergence Mach number results in very close critical Mach number values; thus, Eq. (10) can be used in any case.

The drag divergence Mach number then can be estimated from the Korn equation extended by Mason[12] to include sweep using simple sweep theory. The result is given by

$$M_{\text{DD}} = \kappa_a / \cos\Lambda - (t/c)/\cos^2\Lambda - C_L/10\cos^3\Lambda \tag{11}$$

This model estimates the drag divergence Mach number as a function of an airfoil technology factor κ_a, the thickness-to-chord ratio t/c, the lift coefficient C_L, and the sweep angle at quarter chord, Λ. The airfoil technology factor has a value of 0.87 for a NACA 6-series airfoil section and a value of 0.95 for a supercritical section.[12] If Eq. (11) is substituted into Eqs. (9) or (10), then

$$M_{\text{crit}} = \kappa_a / \cos\Lambda - (t/c)/\cos^2\Lambda - C_L/10\cos^3\Lambda - 0.1 \tag{12}$$

Grasmeyer[10] has used a modified version of the Lock's equation for a transonic strut-braced wing concept. When average thickness/chord ratio of the wing is used in Eqs. (9) and (10), it is possible to obtain good results.[3] Thus, Lock's empirically derived drag rise equation becomes

$$\Delta C_{D_C} = 20\Gamma^4 + (8\Gamma^3/\cos^3\Lambda)C_L + (1.2\Gamma^2/\cos^6\Lambda)C_L^2 + (0.08\Gamma/\cos^9\Lambda)C_L^3 + (0.002/\cos^{12}\Lambda)C_L^4 \tag{13}$$

where

$$\Gamma = M + 0.1 - \kappa_a/\cos\Lambda + (t/c)/\cos^2\Lambda \tag{14}$$

is the compressibility parameter. Equation (13) leads to a quartic drag polar for the flight Mach numbers above critical,

$$C_D = C_{D_{0_c}} - K_{1_c} C_L + K_{2_c} C_L^2 + K_{3_c} C_L^3 + K_{4_c} C_L^4 \tag{15}$$

where

$$C_{D_{0_c}} = C_{D_{\min}} + K C_{L_{\min}}^2 + 20\Gamma^4$$

$$K_{1_c} = 2K C_{L_{\min}} - 8(\Gamma/\cos\Lambda)^3, \qquad K_{2_c} = K + 1.2(\Gamma/\cos^3\Lambda)^2$$

$$K_{3_c} = 0.08(\Gamma/\cos^9\Lambda), \qquad K_{4_c} = 0.002/\cos^{12}\Lambda \tag{16}$$

Equation (16) implies that the drag polar coefficients are functions of the Mach number for a given aircraft design.

Constant Altitude–Constant Mach Number Cruise Range

The constant altitude–constant Mach number cruise range of an aircraft can be determined from the differential equation

$$\mathrm{d}R = -\frac{aM}{c_T}\frac{C_L}{C_D}\frac{\mathrm{d}W}{W} \tag{17}$$

Because

$$\mathrm{d}W = (\gamma/2) p M^2 S\, \mathrm{d}C_L \tag{18}$$

for constant Mach number level flight, then by substituting Eqs. (3) and (18) in Eq. (17)

$$\mathrm{d}R = -\frac{aM^{1-\beta}}{c_{T_0}\sqrt{\theta}}\frac{\mathrm{d}C_L}{C_D} \tag{19}$$

If Eq. (19) is integrated from C_{L_0}, the lift coefficient at the beginning of cruise flight, to C_{L_1}, the lift coefficient at the end of cruise, the range becomes

$$R = \frac{aM^{1-\beta}}{c_{T_0}\sqrt{\theta}} \int_{C_{L_1}}^{C_{L_0}} \frac{\mathrm{d}C_L}{C_D} \tag{20}$$

Because W_0 is the aircraft weight at the beginning of the cruise, and W_F is the cruise fuel weight, then

$$C_{L_1} = (1 - \zeta) C_{L_0} \tag{21}$$

where ζ is the fuel weight fraction

$$\zeta = W_F / W_0 \tag{22}$$

Problem Solution

Based on the calculation method initially introduced by Hale[13] for an aircraft with a simple parabolic drag polar, Cavcar and Cavcar[2,3] derived a solution of the constant altitude–constant Mach number cruise range equation for an aircraft with a cambered wing drag polar of

$$C_D = C_{D_0} - K_1 C_L + K_2 C_L^2 \tag{23}$$

Cavcar and Cavcar's solution of Eq. (20) for incompressible flow case was

$$R = \frac{2aM^{1-\beta}}{c_{T_0}\sqrt{\theta}\sqrt{4K_2 C_{D_0} - K_1^2}} \times \tan^{-1}\frac{1}{2}\left\{\frac{\zeta\sqrt{4K_2 C_{D_0} - K_1^2}\, C_{L_0}}{C_{D_0} - [(2-\zeta)/2]K_1 C_{L_0} + (1-\zeta)K_2 C_{L_0}^2}\right\} \tag{24}$$

However, when compressibility effects are considered, the drag polar is a quartic equation as given by Eq. (15) and will result in a cruise range solution different from Eq. (24).

Solution of the cruise range equation given by Eq. (20) requires numerical treatment in the case of a quartic drag polar. However, a quartic drag polar can be converted into a parabolic drag polar in the form of Eq. (23) by the Weierstrass approximation theorem, which states that any continuous function on a closed and bounded interval can be uniformly approximated on that interval by polynomials to any degree of accuracy (see Ref. 14). Therefore, for the $\lfloor C_{L_0}, C_{L_1} \rfloor$ interval, Eq. (15) drag polar reduces to Eq. (23) form, where the drag polar coefficients are, for $M > M_{\text{crit}}$,

$$C'_{D_0} = C_{D_{\min}} + KC^2_{L_{\min}} + 20\Gamma^4 + 0.004(2-\zeta)(10-10\zeta+\zeta^2)C^3_{L_0} \times \frac{\Gamma}{\cos^9\Lambda} + \frac{0.006}{35\cos^{12}\Lambda}(35-70\zeta+48\zeta^2-13\zeta^3+\zeta^4)C^4_{L_0}$$

$$K'_1 = 2KC_{L_{\min}} - 8\left(\frac{\Gamma}{\cos\Lambda}\right)^3 + \frac{0.24\Gamma}{5\cos^9\Lambda}(5-5\zeta+\zeta^2)C^2_{L_0} + \frac{0.008}{35\cos^{12}\Lambda}(70-105\zeta+51\zeta^2-8\zeta^3)C^3_{L_0}$$

$$K'_2 = K + 1.2\left(\frac{\Gamma}{\cos^3\Lambda}\right)^2 + 0.12\frac{\Gamma}{\cos^9\Lambda}(2-\zeta)C_{L_0} + \frac{0.012}{7\cos^{12}\Lambda}(7-7\zeta+2\zeta^2)C^2_{L_0} \tag{25}$$

and for $M < M_{\text{crit}}$,

$$C'_{D_0} = C_{D_{\min}} + KC^2_{L_{\min}}, \qquad K'_1 = 2KC_{L_{\min}}, \qquad K'_2 = K \tag{26}$$

On the other hand, both Torenbeek[4] and Ojha[15] stated that the angle defined by the arctangent term in the constant altitude–constant speed range equation is usually small and that the arctangent term can be set equal to its argument for optimum cruising. These two approximations result in the following constant altitude–constant Mach number cruise range equation:

$$R = \frac{a}{c_{T_0}\sqrt{\theta}} \frac{\zeta C_{L_0} M^{1-\beta}}{C'_{D_0} - [(2-\zeta)/2]K'_1 C_{L_0} + (1-\zeta)K'_2 C^2_{L_0}} \tag{27}$$

Maximum Range

For constant altitude, maximization of the range must be satisfied by the

$$\frac{\partial R}{\partial M} = 0 \tag{28}$$

condition, which results in

$$(1+\beta)C'_{D_0} + M\frac{\partial C'_{D_0}}{\partial M} - \frac{2-\zeta}{2}\left[(\beta-1)K'_1 + M\frac{\partial K'_1}{\partial M}\right]C_{L_0} + (1-\zeta)\left[(\beta-3)K'_2 + M\frac{\partial K'_2}{\partial M}\right]C^2_{L_0} = 0 \tag{29}$$

from Eq. (27). However, there is no simple analytical solution of Eq. (29) to find the maximum range Mach number. On the other hand, numerical treatments of Eqs. (24) and (27) always result in maximum range Mach number values close to the drag divergence Mach number M_{DD} of the initial cruise conditions. Therefore, for conceptual or preliminary design studies the maximum range Mach number can be set equal to the drag divergence Mach number, $M_{\text{CR}} \approx M_{\text{DD}}$. In this case, the lift coefficient at the beginning of cruise will be

$$C_{L_0} = 2W_0 \big/ \gamma p M^2_{\text{DD}} S \tag{30}$$

Now, use M_{DD} so that M_{CR} can be determined by Eqs. (11) and (30), resulting in

$$M^3_{\text{DD}} - \left(\frac{\kappa_a}{\cos\Lambda} - \frac{t/c}{\cos^2\Lambda}\right)M^2_{\text{DD}} + \frac{W_0/S}{5\gamma p\cos^3\Lambda} = 0 \tag{31}$$

One of the real roots of Eq. (31) gives the drag divergence Mach number,

$$M_{\text{DD}} = \tfrac{1}{3}\left[\kappa_a/\cos\Lambda - (t/c)/\cos^2\Lambda\right]\left[1 + 2\cos\left(\tfrac{1}{3}\cos^{-1} \times \left\{1 - (2.7/\gamma p)(W_0/S)[\kappa_a - (t/c)/\cos\Lambda]^{-3}\right\}\right)\right] \tag{32}$$

In this case, the predicted range from Eq. (27) becomes

$$R(M_{\text{DD}}) = \frac{a}{c_{T_0}\sqrt{\theta}} \frac{2\zeta}{\gamma p} \frac{1}{M^{1+\beta}_{\text{DD}}} \frac{W_0/S}{C_D(M_{\text{DD}}) - \Delta C_{D_{\text{CR}}}(M_{\text{DD}})} \tag{33}$$

where

$$C_D(M_{\text{DD}}) = C_{D_{\min}} + \frac{\left(C_{L_0} - C_{L_{\min}}\right)^2}{\pi A\!\!\!R e\sqrt{1-(M_{\text{DD}}-0.1)^2}} + 0.002 \tag{34}$$

and

$$\Delta C_{D_{\text{CR}}}(M_{\text{DD}}) = \zeta C_{L_0}\left\{\frac{\left(C_{L_0} - C_{L_{\min}}\right)}{\pi A\!\!\!R e\sqrt{1-(M_{\text{DD}}-0.1)^2}} + \frac{0.002}{\cos^3\Lambda}\left[2 - \left(\frac{\zeta C_{L_0}}{\cos^3\Lambda}\right)^2 + \frac{13}{35}\left(\frac{\zeta C_{L_0}}{\cos^3\Lambda}\right)^3\right]\right\} \tag{35}$$

Bréguet Range

The Bréguet range, constant speed–constant lift coefficient cruise is not a practical cruise flight condition due to continuous change of altitude as the fuel is burnt, and altitude changes are usually not allowed by air traffic management systems during enroute flight. However, Kücheman[16] stated that the Bréguet range is an abstract concept and may be regarded as a figure of merit of the whole aircraft. A typical Bréguet range equation

$$R_B = (a/c_T)[M(L/D)]\,\ell n[1/(1-\xi)] \tag{36}$$

implies that ML/D must be maximum for maximum cruise range, when variation of c_T with the Mach number and cruise altitude are ignored. For an aircraft in transonic cruise flight, calculation of maximum ML/D requires numerical treatment of Eq. (15) quartic drag polar. According to Shevell,[6] in cruise, airplanes almost never fly continuously at lift coefficients below the value for minimum C_D nor above $C_L = 0.7$, and flight test curves of C_D vs C_L are fitted with parabolic curves between lift coefficients of about 0.2 and 0.7. Because Bréguet cruise takes place at constant lift coefficient, rather than varying lift coefficients in the case of constant altitude–constant Mach number flight, the quartic drag polar of Eq. (15) can be reduced to a parabolic drag polar around the design cruise lift coefficient. Consider design cruise lift coefficient C_{L_D} and two other lift coefficients, $C_{L_D} + 0.1$ and $C_{L_D} - 0.1$. Equating an approximate parabolic drag polar as given Eq. (23) and the quartic drag polar of Eq. (15) at these three lift coefficients will result in

$$C'_{D_0} = C_{D_{\min}} + KC^2_{L_{\min}} + 20\Gamma^4 + 0.08C_{L_D}\left[C^2_{L_D} - 0.01\right](\Gamma/\cos^9\Lambda) + (0.006/\cos^{12}\Lambda)C^2_{L_D}\left[C^2_{L_D} - 0.01\right]$$

$$K'_1 = 2KC_{L_{\min}} - 8(\Gamma/\cos\Lambda)^3 + 0.08\left[3C^2_{L_D} - 0.01\right](\Gamma/\cos^9\Lambda) + (0.004/\cos^{12}\Lambda)C_{L_D}\left[4C^2_{L_D} - 0.01\right]$$

$$K'_2 = K + 1.2(\Gamma/\cos^3\Lambda)^2 + 0.24C_{L_D}(\Gamma/\cos^9\Lambda) + (0.002/\cos^{12}\Lambda)\left[6C^2_{L_D} + 0.01\right] \tag{37}$$

When Shevell's discussion is considered, $C_{L_D} = 0.45$ is a proper value, which is midpoint between 0.2 and 0.7. Because design cruise lift coefficient is a constant, these coefficients are only functions of the Mach number. For a C_D vs C_L at a constant Mach number,

$$[M(L/D)]_{\max} = M(L/D)_{\max} \tag{38}$$

Because

$$(L/D)_{\max} = 1\Big/\left(2\sqrt{K_2' C_{D_0}'} - K_1'\right) \tag{39}$$

for a cambered wing drag polar,[2] then for a constant Mach number

$$[M(L/D)]_{\max} = M\Big/\left(2\sqrt{K_2' C_{D_0}'} - K_1'\right) \tag{40}$$

However, Eq. (40) is not a global $(ML/D)_{\max}$, and finding the global $(ML/D)_{\max}$ requires

$$\frac{\mathrm{d}}{\mathrm{d}M}\left(\frac{M}{2\sqrt{K_2' C_{D_0}'} - K_1'}\right) = 0 \tag{41}$$

Finding the Mach number that satisfies global $(ML/D)_{\max}$ through Eq. (41) requires numerical methods.

Application

To demonstrate the accuracy of the approximations, proposed solutions are applied to the B747, taking into consideration published and available characteristics of the aircraft. B747 aircraft parameters are presented in Table 1. According to McCormick,[8] the estimated flat-plate area of B747-100 aircraft is 9.29 m^2. However, the drag polar of B747 cited by Andreu,[17] referring to Boeing aircraft, implies a flat-plate area much closer to 8.50 m^2, and so this value is chosen for the application. Resources of the other data values are also given in Table 1 (Refs. 8, 12, 18, and 19). Various range calculations are compared by the introduction of a dimensionless range

Table 1 Aircraft parameters

Parameter	Value
Aerodynamic data	
Clean equivalent flat-plate area	8.50 m^2
Wing area[18]	510.97 m^2
Wing aspect ratio[18]	6.98
Oswald efficiency factor[8]	0.7
Average thickness ratio[19]	9.4%
Airfoil technology factor[12]	0.89
Sweepback at quarter chord[18]	37.5 deg
Weight data	
Maximum takeoff weight[18]	333,400 kg
Maximum fuel weight[18]	147,181 kg
Taxi out, takeoff, and climb fuel ratio to maximum fuel weight	10%
Cruise fuel ratio to maximum fuel weight	80%
Flight condition	
Cruise altitude	37,000 ft
Propulsion data	
TSFC Mach number exponent	0.5

$$R' = \left(c_{T_0}\big/a_t\right)\sqrt{\theta_t}\,R \tag{42}$$

where subscript t indicates the values at the tropopause.

Figure 3 shows dimensionless range calculated by Eq. (24) vs Mach number for various altitudes, when $\beta = 0$. According to Fig. 3, optimum constant altitude–constant Mach number flight takes place at 37,000 ft and $M = 0.86$. Both numerically and approximately calculated Bréguet ranges are also given in Fig. 3. Maximum Bréguet range is also achieved at $M = 0.86$, although Bréguet cruise occurs at varying altitudes starting from 31,000 ft and ending at 41,000 ft. Figure 3 also shows both numerical and approximate calculations of Bréguet range are close to the constant altitude–constant Mach number cruise at 37,000 ft. The difference between the numerical Bréguet range solution and constant altitude–constant Mach number cruise at 37,000 ft is found to be less than 1% for all Mach numbers and 0.26% at $M = 0.86$. Even the difference between the approximate Bréguet range and constant altitude–constant Mach number cruise at 37,000 ft is found to vary from 0.5 to 1.7% for all Mach numbers and is only 0.81% at $M = 0.86$.

Figure 4 presents dimensionless range vs Mach number for the $\beta = 0.5$ condition. The increased dimensionless range values and decreased maximum range Mach number, now about 0.85, imply the importance of the dependence of thrust specific fuel consumption (TSFC) on Mach number. The value $\beta = 0.5$ representing a turbofan engine results in superior range performance, but with a slower cruise Mach number.

Both Figs. 3 and 4 also show the accuracy of the proposed Bréguet range approximate solution. By the selection of $C_{L_D} = 0.45$, an approximate Bréguet solution with an error of less than 1% is obtained.

Figure 5 shows further detail of the range calculations for 37,000 ft. In this case, calculations by approximate equation (27) are also included. Equation (27) predicts the dimensionless range maximum 5% longer than the exact equation (24). For 37,000-ft optimum altitude, it is found that flight with M_{DD} results in only less than 1% reduction in range compared to the maximum range. However, flight with M_{DD} at the altitudes other than optimum results in

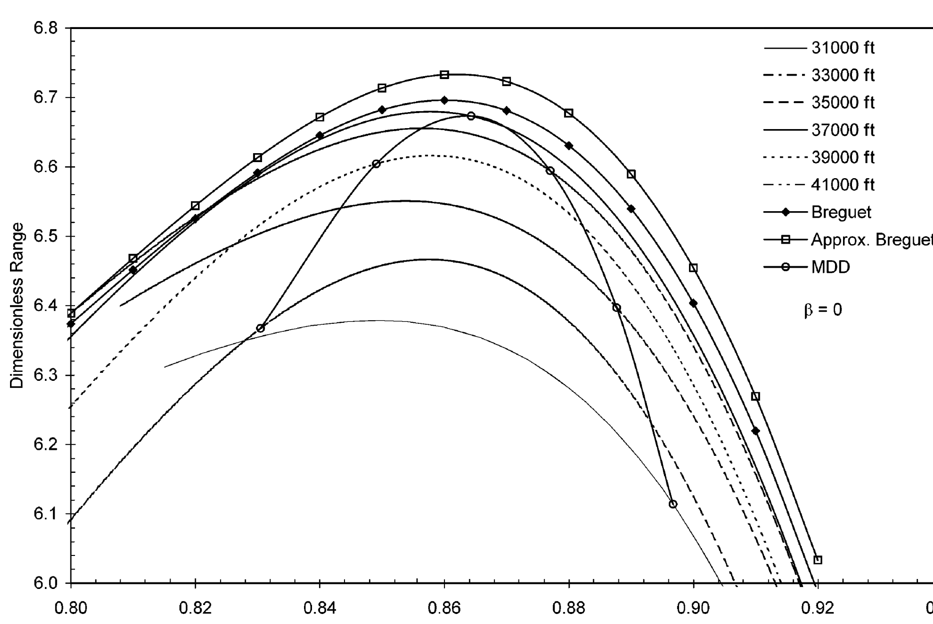

Fig. 3 Dimensionless range vs Mach number at $\beta = 0$.

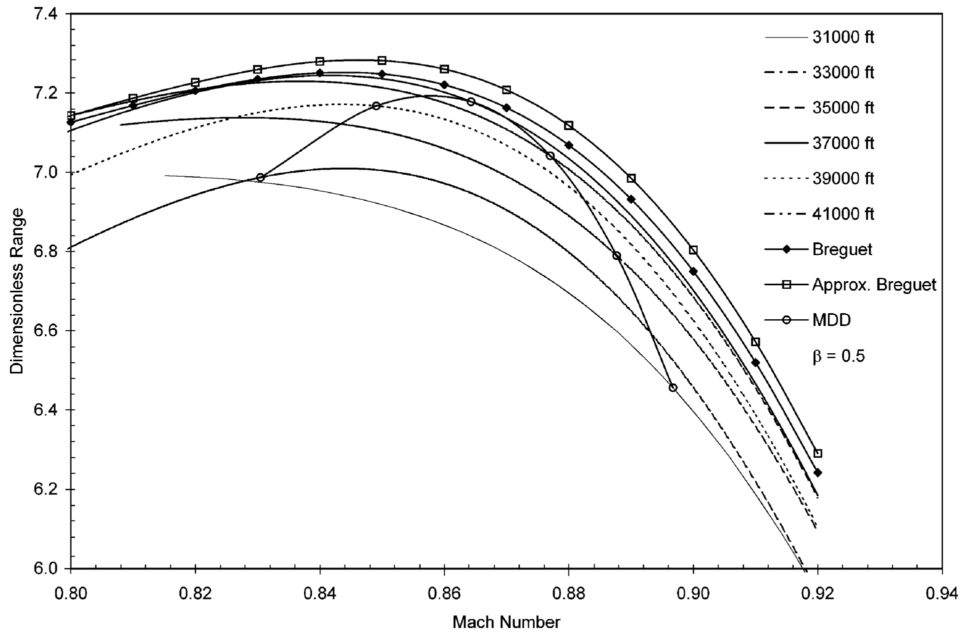

Fig. 4 Dimensionless range vs Mach number at $\beta = 0.5$.

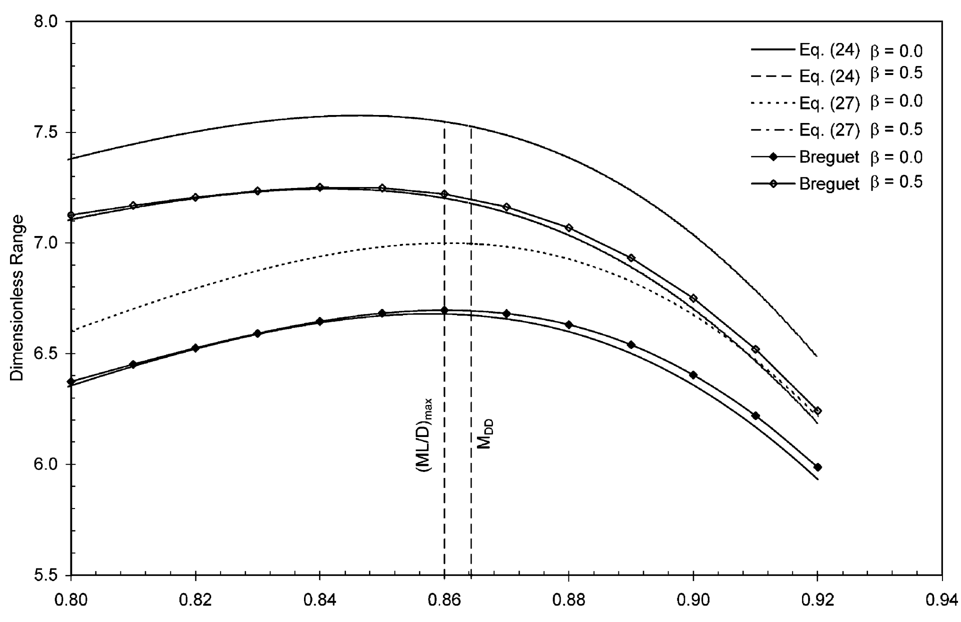

Fig. 5 Dimensionless range vs Mach number at 37,000 ft.

much shorter ranges than maximum, as shown in Figs. 3 and 4. For example, flight with M_{DD} causes 8% reduction in range at 31,000 ft for $\beta = 0.5$.

Figure 5 also shows that the drag divergence Mach number and the Mach number for $(ML/D)_{\max}$ is almost same at the optimum altitude. When compared to the well-known $1.32V_{\mathrm{MD}}$ or $1.32M_{\mathrm{MD}}$ maximum range cruise speed for incompressible flow case, this ratio is found to be $1.036M_{\mathrm{MD}}$ for the compressible case if it is assumed that M_{DD} is the economical cruise speed. This is shown in Fig. 6. It is seen that actual maximum range Mach number is much closer to the minimum drag both for $\beta = 0$ and $\beta = 0.5$.

According to the thrust and specific fuel consumption data for the Pratt and Whitney JT9D-7A turbofan engines presented by McCormick,[8] TSFC is about 0.7 for the altitudes above 35,000 ft at $M = 0.86$. If this value is applied to the calculations, cruise range of the aircraft is found to be approximately 5450 n miles (10,093 km) by Eq. (24) and 5700 n miles (10,556 km) by Eq. (27). The range payload diagram presented by Boeing[20] for the B747-200 aircraft shows a 5200-n miles (9630 km) range for step altitude cruise from 31,000 to 39,000 ft and with Federal Aviation Regulations and Air Transport Association reserves. Calculations presented here only consider 10% of total fuel for taxi-out and climb and 10% of total fuel for descent, approach, and landing. Therefore, calculated range by approximate method is close to real aircraft data. A summary of the calculation results is given in Tables 2 and 3.

Table 2 Mach number calculation results for 37,000 ft

Important Mach numbers	Value
Minimum drag	0.834
For $(ML/D)_{\max}$	0.860
Drag divergence, M_{DD}	0.864

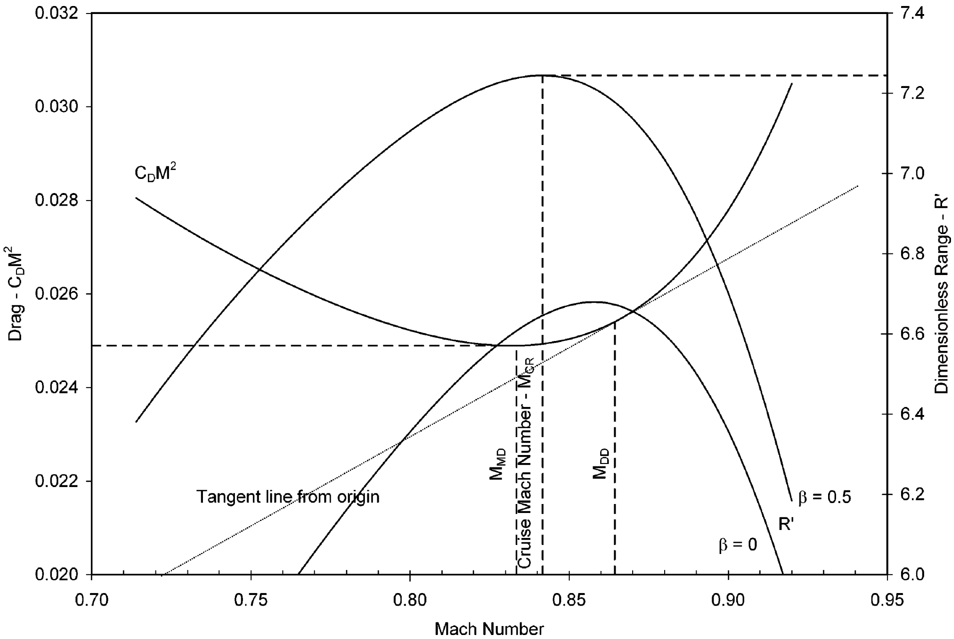

Fig. 6 Minimum drag and dimensionless range versus Mach number.

Table 3 Range calculation results for 37,000 ft

Range calculations	Exact equation (24)	Approximate equation (28)
$\beta = 0.5$		
Maximum dimensionless range	7.244	7.575
Maximum range Mach number	0.842	0.846
Dimensionless range at M_{DD}	7.178	7.526
$\beta = 0$		
Maximum dimensionless range	6.680	6.999
Maximum range Mach number	0.858	0.860
Dimensionless range at M_{DD}	6.673	6.997

Conclusions

A quick assessment of the cruise range and cruise conditions is a key task in the conceptual or preliminary design phase of an aircraft for initial weight estimations. Although numerical treatment is more common in this new century, accurate analytical approximations are still required for quick analysis. However, the accuracy is obtained by realistic assumptions. When cruise range prediction for preliminary or conceptual design of today's modern transport aircraft is considered, it is impossible to assume a simple parabolic drag polar and to neglect compressibility effects, even for long-range cruise conditions. Calculations here show that it is possible to reach an analytical solution by considering a realistic drag polar, specific fuel consumption, and compressibility effects, although some assumptions still exist.

The method presented results in highly accurate range prediction, as shown with the application to Boeing 747 aircraft. However, accuracy of the method is strongly dependent on the correct assumption of the airfoil technology factor. Because of the simplicity of the method, the range calculation presented here can be simply incorporated into multidisciplinary design optimization calculations as an alternative to the Breguet range.

The method may also be used in aircraft performance and aircraft design teaching to provide candidate engineers a more realistic case.

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